Statistical Modeling for the Minimum Standby Supply Voltage of a Full SRAM Array

Jiajing Wang[1],

Amith Singhee[2],

Rob A. Runtenbar[2],

Benton H. Calhoun[1]

[1]University of Virginia, Charlottesville, VA

[2]Carnegie Mellon University, Pittsburg, PA

Outline

- **Motivation**
- **SNM and DRV**
- **New DRV Model Based on SNM**
- **DRV Model Evaluation**
- **Conclusion**

Motivation I:
SRAM Leakage Power Savings & Stability

- **Leakage power increases with scaling**
- **SRAM leakage power dominates**
- **Standby supply voltage (V_{DD}) scaling reduces leakage power effectively**
- **SRAM stability is degraded with V_{DD} scaling**
- **Data Retention Voltage (DRV):**

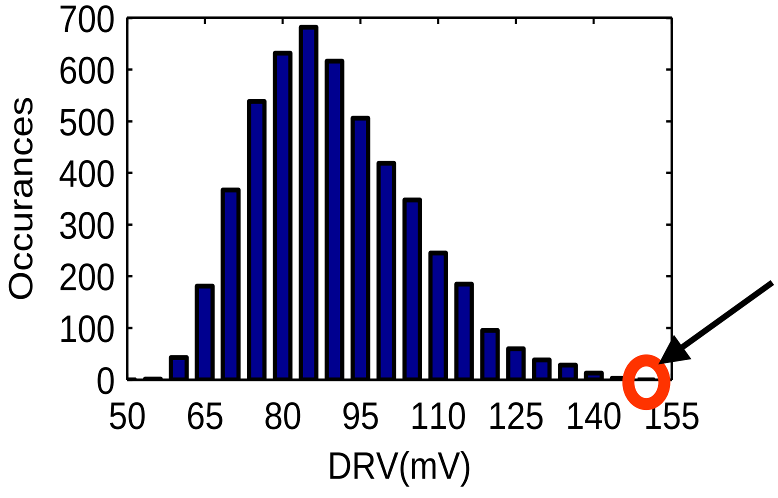
  min(V_{DD}) for preserving cell state

Motivation II: DRV Distribution of a Full SRAM

- **Variations impact DRV of different cells**
  - V_T variation has the strongest impact
  - DRV is distributed on the same die
  - DRV distribution has a heavier tail on right

- **The tail is very important**

e.g. Histogram from a 5k-point Monte-Carlo (M-C) in 90nm



Tail sets min(V_{DD}) for the whole SRAM

Motivation III: Methods for DRV Tail Estimation

- **An analytical model [*Qin et al, ISQED2004*] for DRV of individual bitcells**
  – Complex and hard to find the tail value
- **Full M-C simulation**
  – Accurate but too expensive for large SRAMs
- **Small M-C simulation + extrapolation**
  – Inaccurate since DRV is a non-Gaussian distribution
  – Neither normal nor log-normal
- **This work: Two fast and accurate methods**
  – A new DRV model based on SNM
  – Statistical Blockade tool [*Singhee et al, DATE2007*]

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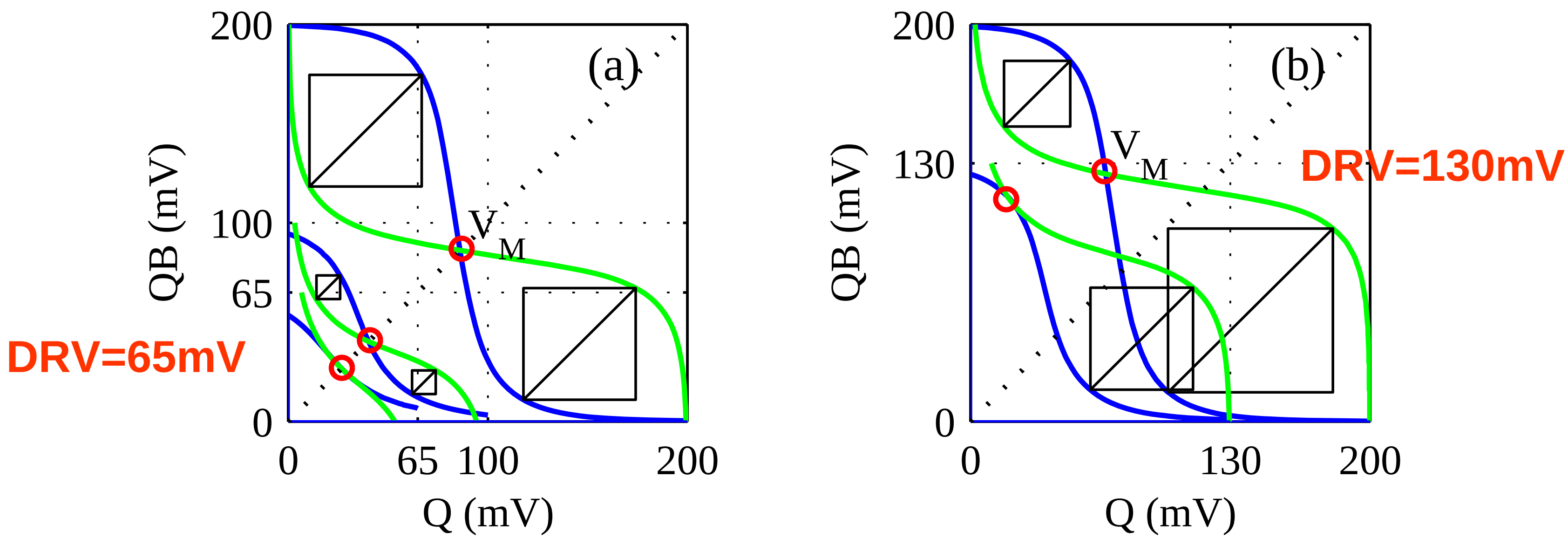
SNM & DRV I

* Butterfly curves of the balanced/imbalanced cell at various V_{DD}

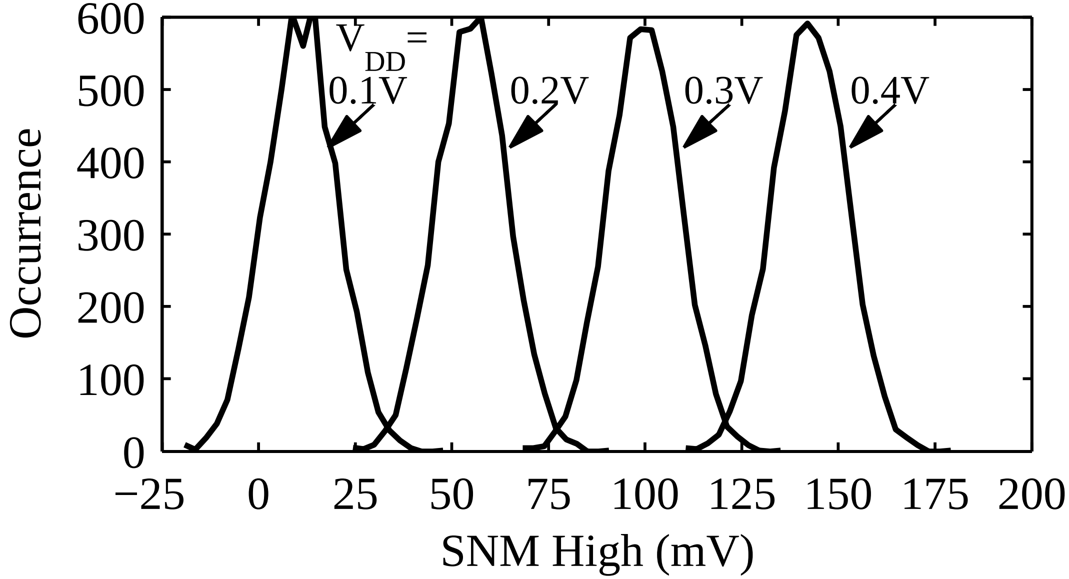


- **Static Noise Margin (SNM) – the metric of stability**
  - SNMH: upper-left square
  - SNML: lower-right square
  - SNM = min(SNMH, SNML)
- **DRV = V_{DD} (when SNM=0)**

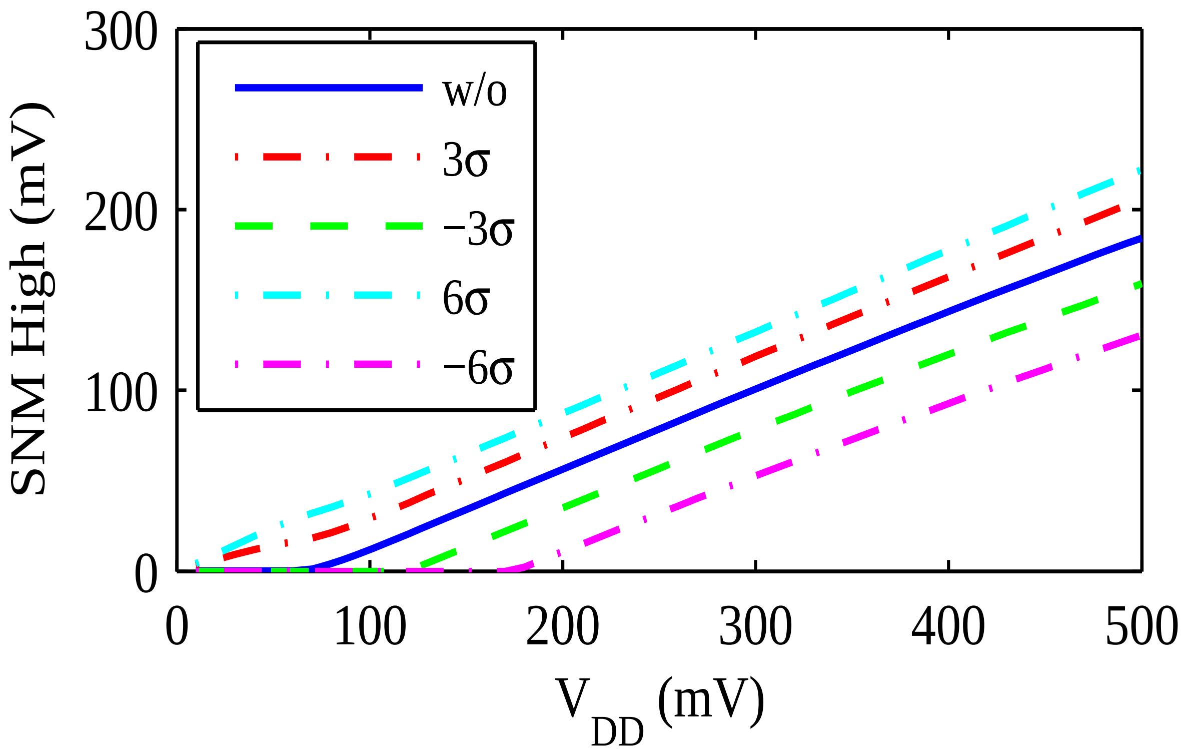
SNM & DRV II

- **SNMH/SNML/SNM statistics**
  - SNMH/SNML is approximately **normally** distributed $N(\mu, \sigma^2)$
  - SNMH & SNML are approximately **identically** distributed
  - SNM statistics can be obtained from SNMH/SNML statistics [*Calhoun et al, JSSC2006*]

- **How the distribution of SNMH changes with V_{DD}**
  - mean value μ moves, but the shape (i.e. σ) almost keeps same



SNM & DRV III



- **SNMH vs. V_{DD} with/without mismatch in 1 FET**
  - almost **linear** before hitting zero (DRV point)
  - approximately **constant slope** regardless of mismatch
  - slope (*k*) extracted from DC sweep simulation

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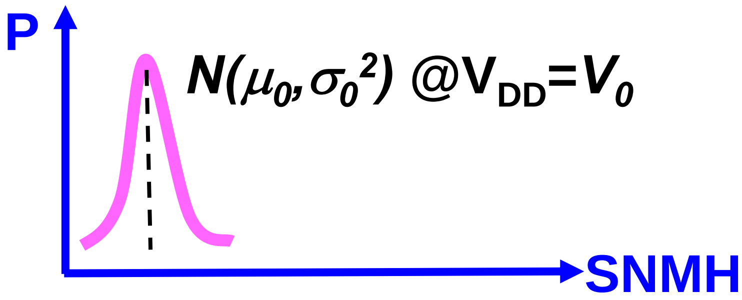
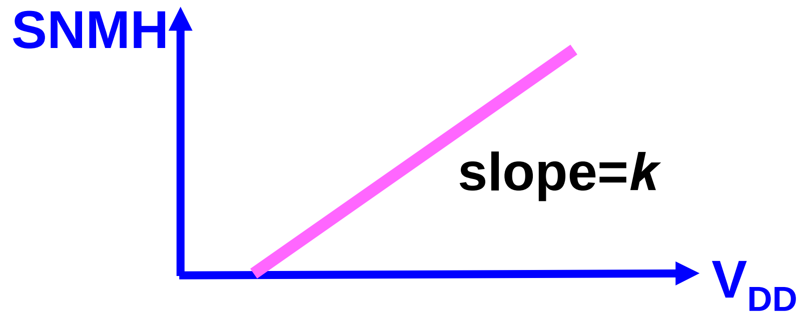
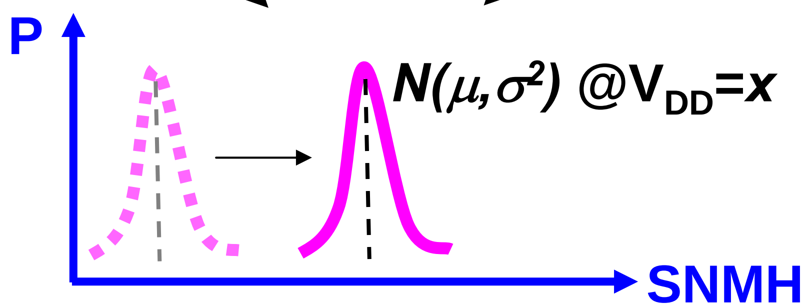
Derive DRV Model from SNM





pick an initial V_{DD} (V_0);
get the statistics (μ_0,σ_0) of SNMH at V_0 from M-C sim

get the linear dependency (k) of SNMH on V_{DD} from DC sweep

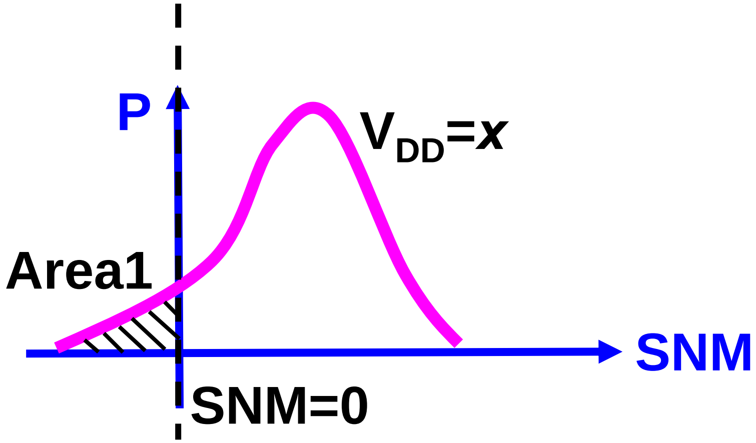


get SNMH statistics (normally distributed) at V_{DD} point *x:*

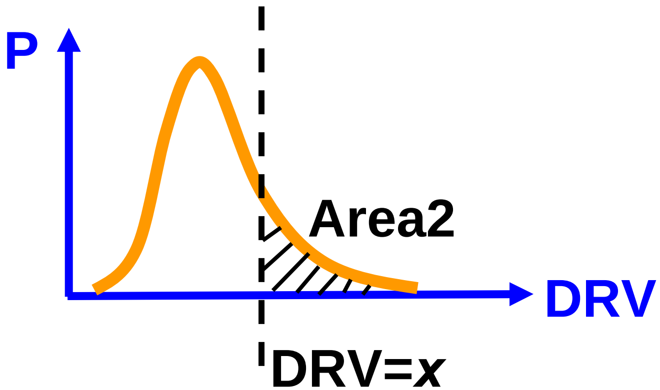
PDF: f_{SNMH} ***and CDF:*** F_{SNMH} **with**

$\mu = \mu_0+k(x-V_0)$; $\sigma = \sigma_0$

Derive DRV Model from SNM



Area1 corresponds to Area2



⇓ ***assume SNMH & SNML are i.i.d.***

SNM = min(SNMH, SNML);

so SNM CDF is (from order statistics):

$$F_{SNM} = 2F_{SNMH} - (F_{SNMH})^2$$

⇓

DRV = V_{DD}(SNM=0)

$$P\left(DRV \le x\right) = 1 - P(SNM \le 0, V_{DD} = x)$$

⇓

get DRV statistics from SNM statistics

SNM & DRV Model

- **CDF of SNM distribution when V_{DD}=*x***

$$P(SNM \le s, V_{DD} = x) = erfc\left(-\frac{s-\mu_0-k(x-V_0)}{\sqrt{2}\sigma_0} \right) - \frac{1}{4}\left(erfc\left(-\frac{s-\mu_0-k(x-V_0)}{\sqrt{2}\sigma_0} \right) \right)^2 \quad \textbf{(1)}$$

- **CDF of DRV distribution**

$$F_{DRV}(x) = 1 - erfc\left(\frac{\mu_0 + k(x-V_0)}{\sqrt{2}\sigma_0} \right) + \frac{1}{4}\left(erfc\left(\frac{\mu_0 + k(x-V_0)}{\sqrt{2}\sigma_0} \right) \right)^2 \quad \textbf{(2)}$$

- **inverse CDF of DRV distribution**

$$F_{DRV}^{-1}(x) = \frac{1}{k}\left(\sqrt{2}\sigma_0 \cdot erfc^{-1}\left(2 - 2\sqrt{x} \right) - \mu_0 \right) + V_0 \quad \textbf{(3)}$$

Parameters

k: the slope of SNMH vs. V_{DD}

V_0: the initial supply voltage for the small M-C sim

μ_0, σ_0: the mean and standard deviation of SNMH when $V_{DD}=V_0$

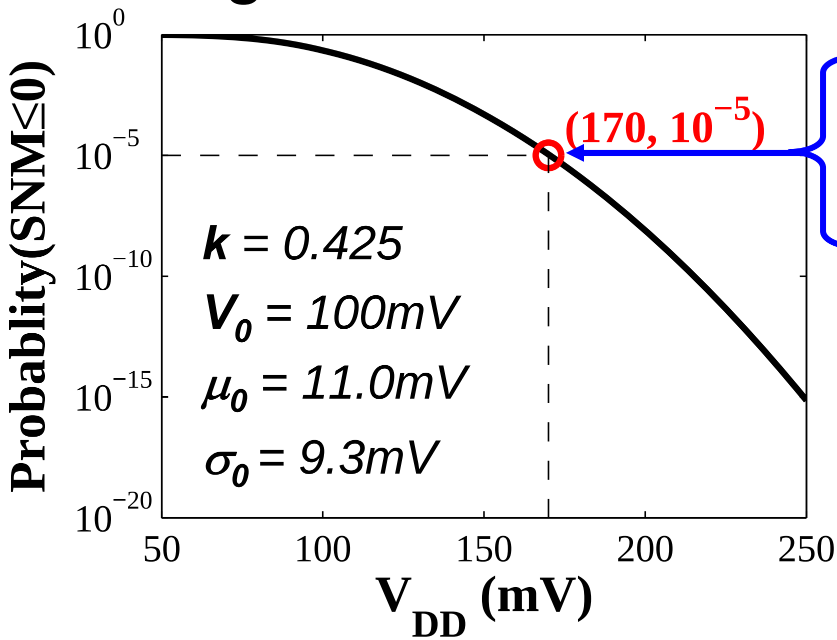
erfc( ): the complementary error function

How to Use Our Model

- **Steps**
  1. extract ***k*** from a DC-sweep of SNM/SNMH vs. V_{DD}
  2. pick $\boldsymbol{V_0}$, extract μ_0 & σ_0 from a 1.5~5K-point M-C simulation for SNMH
  3. use **eq.(1)** to calculate P(SNM≤*s)* at some V_{DD} point *x or*
  4. use **eq.(2)** to calculate P(DRV≤*x) or*
  5. use **eq.(3)** to calculate the V_{DD} that is necessary to ensure that P(DRV≤V_{DD})=*x*

How to Use Our Model

- **One example: Memory failure probability with V_{DD} scaling**



- when V_{DD} =170mV, failure probability is 10^{-5}
- DRV for a 100Kb memory is 170mV

- **If memory must tolerate some noise margin (e.g. 20mV)**
  - Use *s=20mV* in eq.(1)
  - Redefine $DRV=V_{DD}(SNM \leq 20mV)$

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DRV Model Evaluation

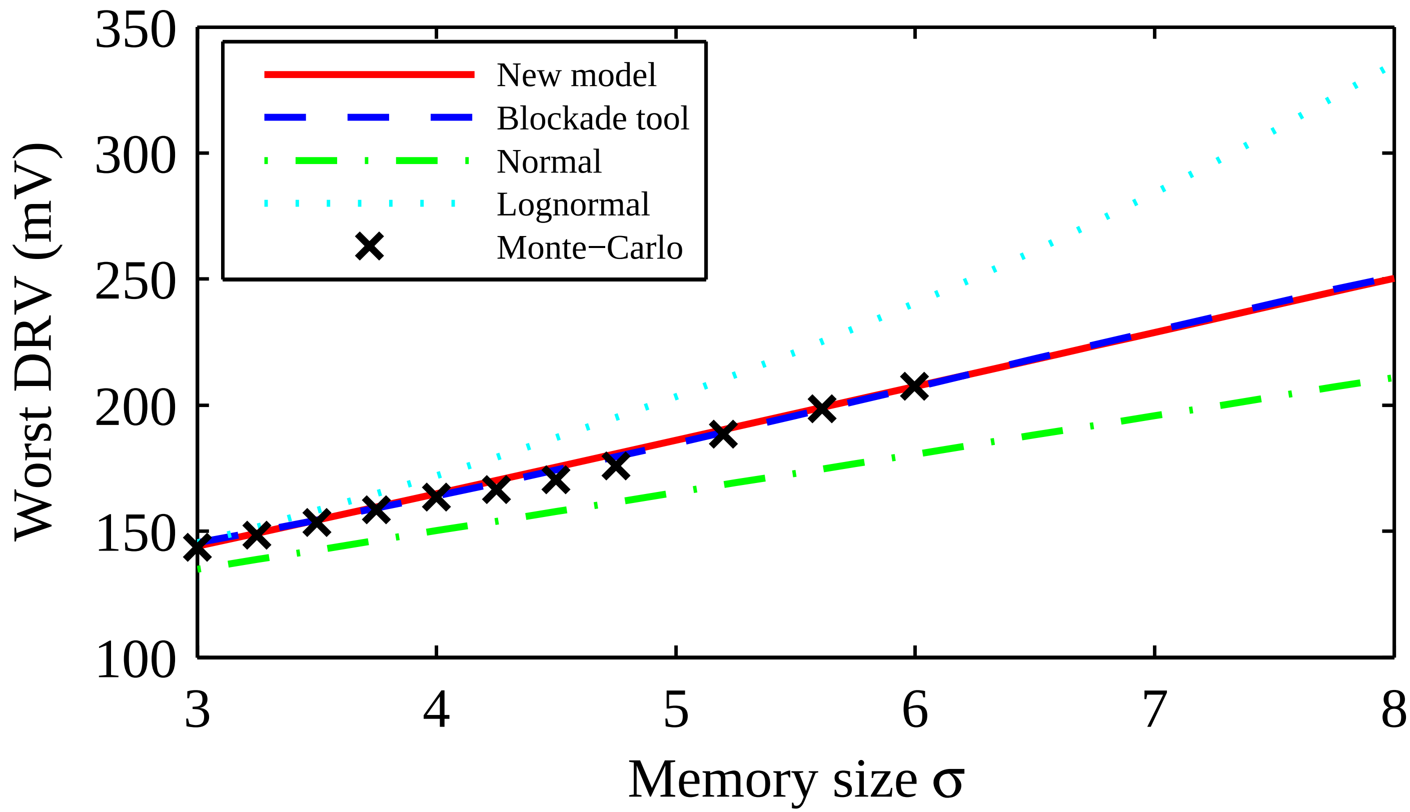
- **Compared with an alternate fast approach, *the Statistical Blockade (SB) tool [Singhee et al, DATE2007]*, up to 8σ**

- **Compared with Monte-Carlo up to 6σ**

SB Tool & its application for DRV

- **SB is a fast M-C simulation tool for rare events**
  - Perform initial sampling
  - Build a classifier to filter samples *prior* to simulation
  - Simulate only those points that are classified as tail points
- **SB also builds a model for tail**
  - Fit true tail points to the Generalized Pareto Distribution (GPD)
  - Estimate longer tails with GPD model
- **Advantage**
  - Generic, i.e. can be used for any tail statistics
  - Succeeded in previous tests on SRAM and flipflop
- **This work: New application to DRV**

DRV Model Comparison



- **Compared with M-C**
  - **Average error rate 1.3% out to 6σ**
  - **10^5× speedup at 6σ**
- **Compared with SB**
  - **Agreement up to 8σ**
  - **10× faster**

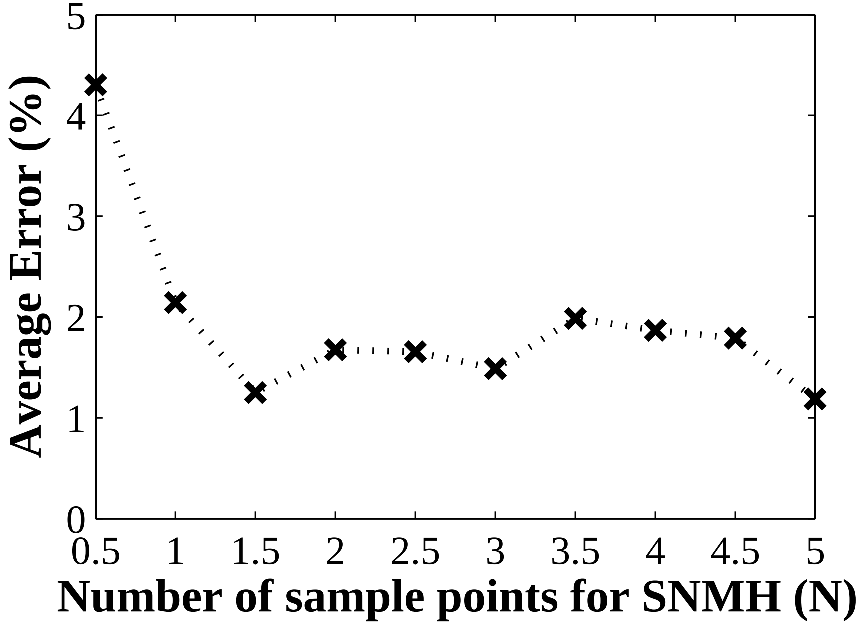
DRV Model Evaluation II

- **Fitting parameters of new model**
  - ***k***: the slope of SNMH vs. V_{DD}
  - $\boldsymbol{V_0}$: the initial supply voltage
  - $\boldsymbol{\mu_0}$: the mean of SNMH when $V_{DD}=V_0$
  - $\boldsymbol{\sigma_0}$: the standard deviation of SNMH when $V_{DD}=V_0$
  - ***N***: the number of M-C sample points for extracting SNMH statistics

- **To what extent does the model depend on parameter selection?**

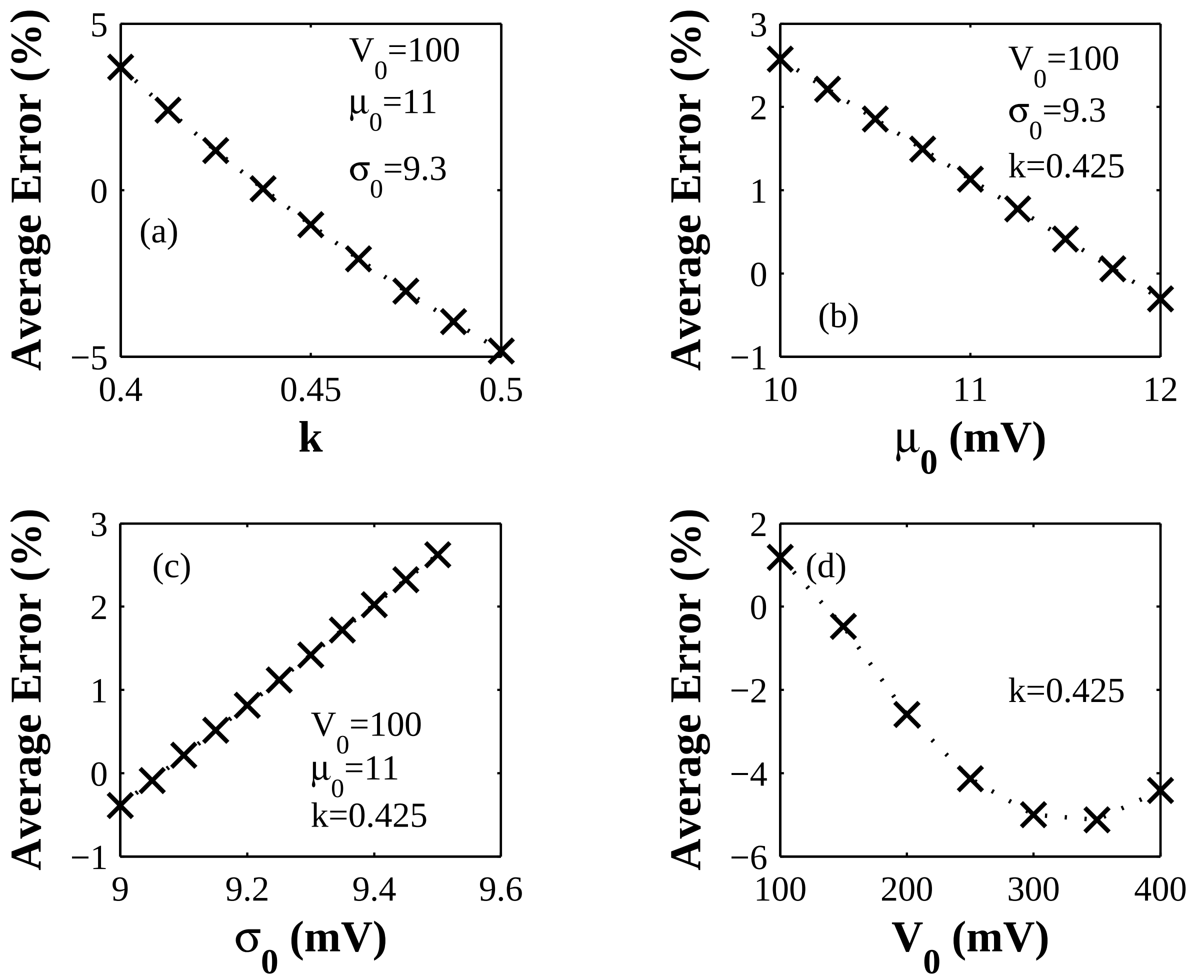
DRV Model Sensitivity I

- **When N changes from 1k to 5k, the average error rate over Monte-Carlo is <3%**



DRV Model Sensitivity II

- When *k*, μ_0, σ_0, or V_0 varies, the average error rate over Monte-Carlo is <6%



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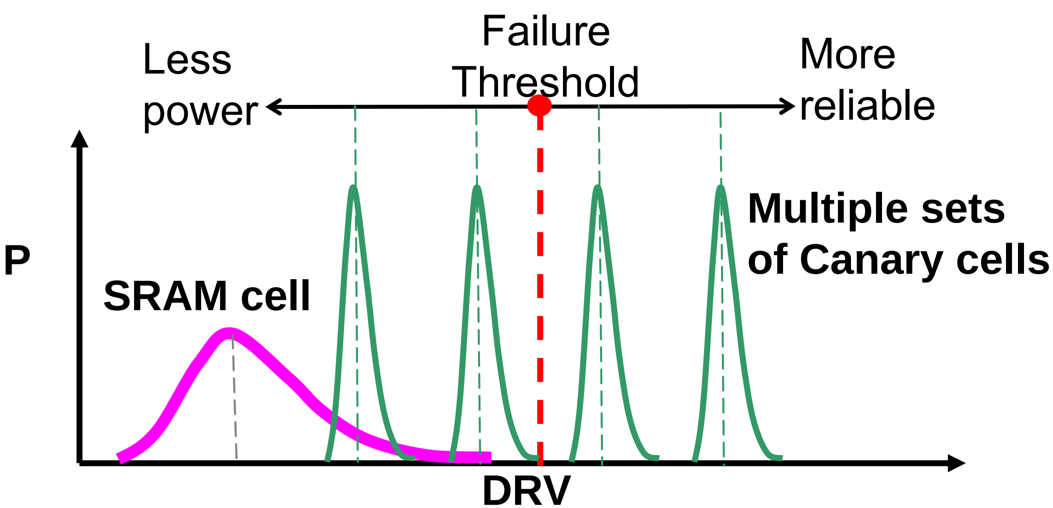
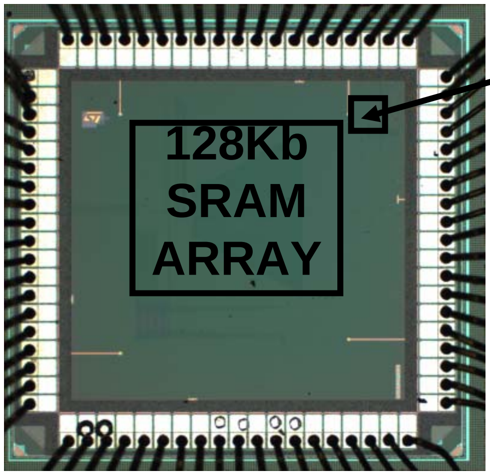
Conclusion

- **DRV of the SRAM cells is distributed**
  - Caused by within-die variations (i.e. mismatch)
  - The tail point determines the minimum V_{DD}
- **Two new methods are proposed to model DRV tail**
  - A new model based on the connection of DRV and SNM
  - The Statistical Blockade tool, applied to DRV for the first time
- **Model accuracy and speed**
  - Compared with M-C (up to 6σ), avg. error rate is **<2%** for model and SB tool
  - New model is highly consistent with SB tool (up to 8σ)
  - New model is insensitive to parameter fluctuations
  - Compared with M-C for 1G-b memory, new model offers **~10^5x** speedup and SB tool offers **~10^4x**

Conclusion

- **Model application: A canary-replica feedback scheme for standby V_{DD} scaling in SRAM *[Wang et al, CICC2007]***
  - Estimate SRAM DRV tail
  - Estimate $P(DRV_{SRAM}<DRV_{canary})$ to configure canary cells
  - Allows closed loop standby power management





Thank You!

Q & A

DRV Reduction Techniques

- **Local mismatch impacts DRV most**
  - Use larger transistor sizes, which reduce the spread of the local threshold voltage variation
- **Global P/N fet strength mismatch also impacts DRV**
  - Move a N/P strong process towards being balanced by using adaptive body biasing
- **Bitline leakage impacts DRV significantly when mismatch occurs on the access transistor**
  - Use negative wordline or floating Bitline, etc., to reduce bitline leakage